CAPS MATCH 2026

Day 1

Time allowed: 4 hours 30 minutes

Each problem is worth 7 points.

No calculators, phones, or other electronic devices are permitted.
Write your solutions clearly and justify all answers.

Problem 1.

In a sequence a_1, a_2, $\ldots$, a_{100} of 100 mutually different real numbers, we say that an index $i \in \{2, 3, \ldots, 99\}$ is *good* if $a_1 + \cdots + a_{i-1} = a_{i+1} + \cdots + a_{100}$. What is the largest possible number of good indices?

Problem 2.

A positive integer k is called *divisive* if there exist only finitely many quadruples (x, y, z, w) of positive integers satisfying

$$x^2 + y^2 + z^2 + w^2 = xyzw \qquad \text{and} \qquad k \nmid x^2 + y^2 + z^2 + w^2.$$

Find all divisive positive integers.

Problem 3.

Let $ABCD$ be a quadrilateral possessing an incenter I. Points K and L are chosen on the segments IA and IC, respectively, such that $CK \perp ID$ and $AL \perp IB$. Points P and Q are chosen on the segments AD and BC, respectively, such that $\angle AKP = \angle ICD$ and $\angle CLQ = \angle IAB$. Prove that $IP = IQ$.

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Day 2

Time allowed: 4 hours 30 minutes

Each problem is worth 7 points.

No calculators, phones, or other electronic devices are permitted.
Write your solutions clearly and justify all answers.

Problem 4.

Tom and Jerry play the following game on a 16×16 chessboard. Jerry starts by selecting some, but not all of the squares, and placing one red pebble on each of the selected squares. Tom then places his one black pebble on an empty square and chooses whether he wants to be the hunter or the prey. After that, Tom and Jerry alternate turns with Jerry starting. In each turn, a player chooses one of his pebbles and moves it to a square sharing an edge with the current square. If such a turn would move the pebble outside of the board, the pebble appears on the other side of the board (i.e. if the pebble is on the left border, it appears in the same row on the right border and similarly for the other cases). If at any point in the game a pebble of the hunter is on the same square as a pebble of the prey, the hunter wins and the game ends. An infinite game is a win for the prey. Which of the two players has a winning strategy?

Problem 5.

Let M be the midpoint of side AD of a cyclic quadrilateral $ABCD$. Points K and L are chosen on the rays MB and MC respectively such that $\angle DAK + \angle ACM = \angle ABM$ and $\angle LDA + \angle MBD = \angle MCD$. Prove that the points B, C, K, and L lie on one circle.

Problem 6.

Let $n \geq 2$ be an integer and let $a_1, a_2, \ldots, a_n$ be non-zero real numbers such that

$$a_1 + \frac{1}{a_2}, \quad a_2 + \frac{1}{a_3}, \quad \ldots, \quad a_{n-1} + \frac{1}{a_n}, \quad a_n + \frac{1}{a_1}$$

are all integers. Prove that

$$a_1 a_2 \ldots a_n + \frac{1}{a_1 a_2 \ldots a_n}$$

is also an integer.